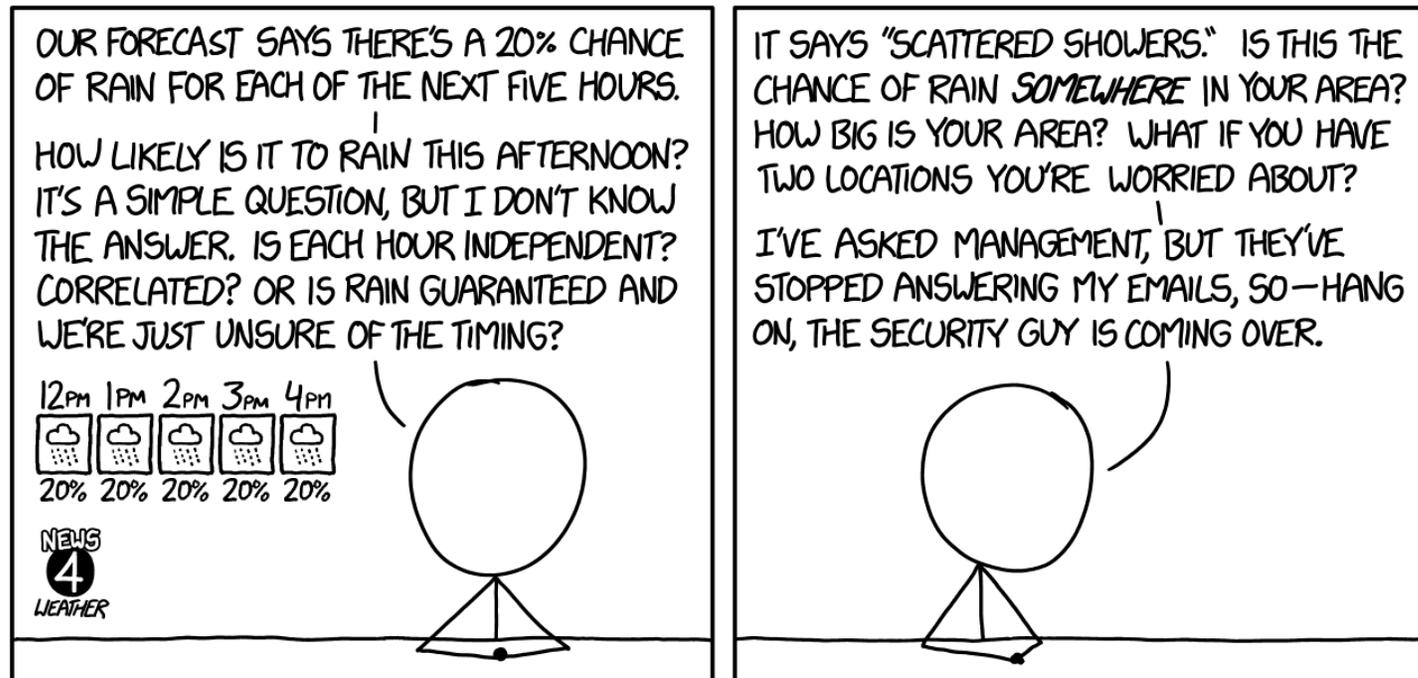


Stat 88: Probability and Mathematical Statistics in Data Science



<https://imgs.xkcd.com/comics/meteorologist.png>

Lecture 1: 1/17/2024

Course introduction and the basics

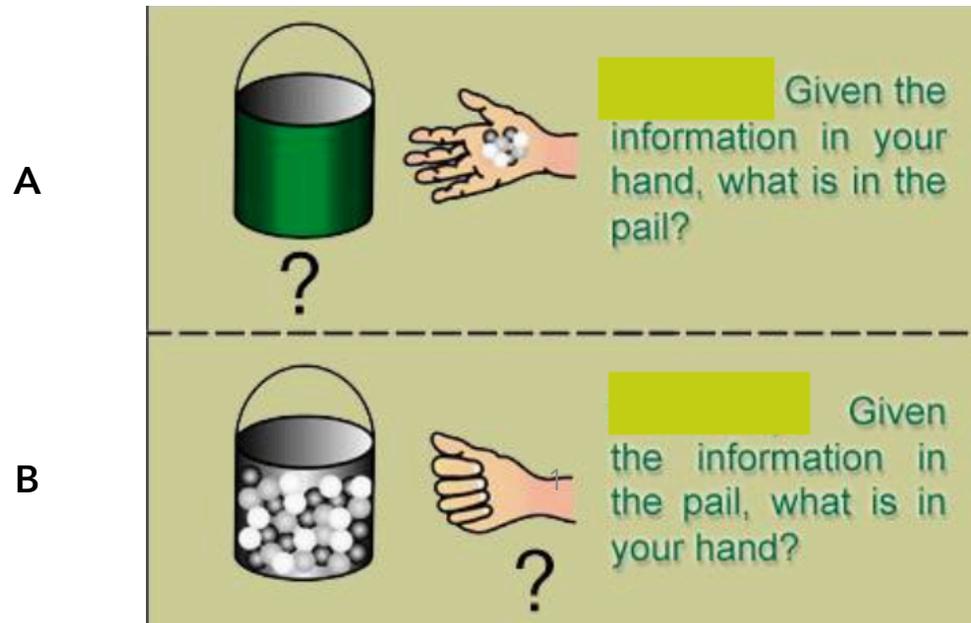
Shobhana Stoyanov

Agenda

- Course resources:
 - Course site: <http://stat88.org>
 - Announcements and discussions: [Ed discussion forum](#)
 - Assignments and grades: [Gradescope](#)
- Put your questions about the course and today's lecture on the thread for Lecture 1
- Introduce yourself to two people sitting near you, tell them your name, where you were born, and what you would be famous for, if you were famous.
- The Basics:
 - terminology
 - assumptions
 - proportions
 - distribution

Probability vs Statistics

- Discuss which is probability and which is statistics:



Section 1.1.1: Basic vocabulary or terminology

- The act of shuffling a deck and then drawing a card has an element of chance - you won't always get the same card.
- Any activity that has chance associated with it is called an **experiment** or a random experiment if there is exactly one of several possible **outcomes** or results, and chance or randomness is involved - that is, each time we perform the action, the outcome will be different, and we don't know exactly which outcome will occur.
- Which of the following are experiments?
 - Roll a pair of dice
 - Read your textbook
 - Buy a raffle ticket
 - Draw 52 cards from a standard deck, without **replacement**.
- An **event** is a description of the result, and might include several outcomes. For example, rolling a die and having the sum of the rolls be 4.

Cards

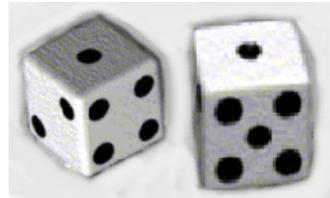
Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades

	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

- If you have a well-shuffled deck of cards, and deal 1 card from the top, what is the chance of it being the queen of hearts? What is the chance that it is a queen (any suit)? What assumptions are you making?
- If you deal 2 cards, what is the chance that at least *one* of them is a queen? How do these relate to populations and samples?

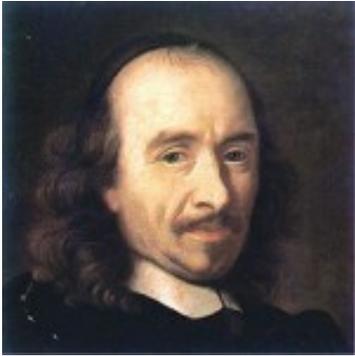
De Méré's Paradox

- We can think about probability as a numerical measure of uncertainty, and we will define some basic principles for computing these numbers.
- These basic computational principles have been known for a long time, and in fact, gamblers thought about these ideas a lot. Then mathematicians investigated the principles.
- Famous problem: will the probability of **at least one six** in **four** throws of a die be equal to prob of **at least a double six** in 24 throws of a pair of dice.
- Note: single = die, plural = dice:



Origins of probability: de Méré's paradox

Questions that arose from gambling with dice.



Antoine Gombaud,
Chevalier de Méré



Blaise Pascal



Pierre de Fermat



The dice players
Georges de La Tour
(17th century)

Terminology

- **Experiment:** action that results in exactly one of several possible outcomes or results, and chance or randomness is involved - that is, each time we perform the action, the outcome will be different, and we don't know exactly which outcome will occur.
- An *event* is a collection of outcomes.
- A collection of all possible outcomes of an action is called a *sample space* or an *outcome space*. Usually denoted by Ω (sometimes also by S).
- An event is always a subset of Ω . Suppose we call the event A , then we write this as $A \subset \Omega$

Computing probabilities: what do we often assume?

- If you have a well-shuffled deck of cards, and deal 1 card from the top, what is the chance of it being the queen of hearts? What is the chance that it is a queen (any suit)?
- How did you do this? What were your assumptions?
- Say we roll a die. What is Ω ?
- What is the chance that the die shows a multiple of 3? What were your assumptions?

Chance of a particular outcome

- We usually think of the chance of a particular outcome (roll a 6, coin lands heads etc) as the number of ways to get that outcome divided by the total possible number of outcomes.

$$\frac{\text{\# of particular outcomes of interest}}{\text{total \# of outcomes possible}}$$

- So if A is an event (subset of Ω), then $P(A) = \frac{\#(A)}{\#(\Omega)}$, $A \subseteq \Omega$
- If an experiment has a **finite** number of possible *equally likely* outcomes, then the probability of an event is the proportion of outcomes that are included in the event.

Cards

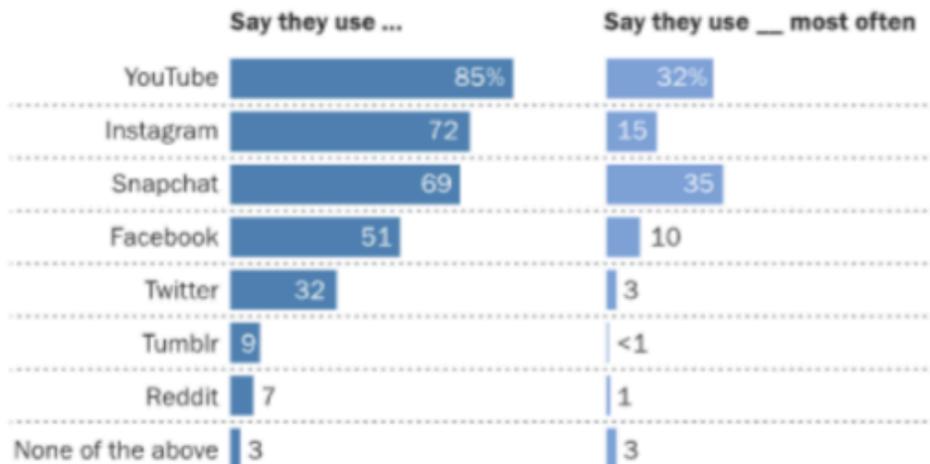
- If you deal 2 cards, what is the chance that at least *one* of them is a queen?

Not equally likely outcomes

- What if our assumptions of equally likely outcomes don't hold (as is often true in life, data is messier than nice examples).
- Here is a graphic from Pew Research displaying the results of a 2018 survey of social media use by US teens.

YouTube, Instagram and Snapchat are the most popular online platforms among teens

% of U.S. teens who ...



Note: Figures in first column add to more than 100% because multiple responses were allowed. Question about most-used site was asked only of respondents who use multiple sites; results have been recalculated to include those who use only one site. Respondents who did not give an answer are not shown.

Source: Survey conducted March 7-April 10, 2018.

"Teens, Social Media & Technology 2018"

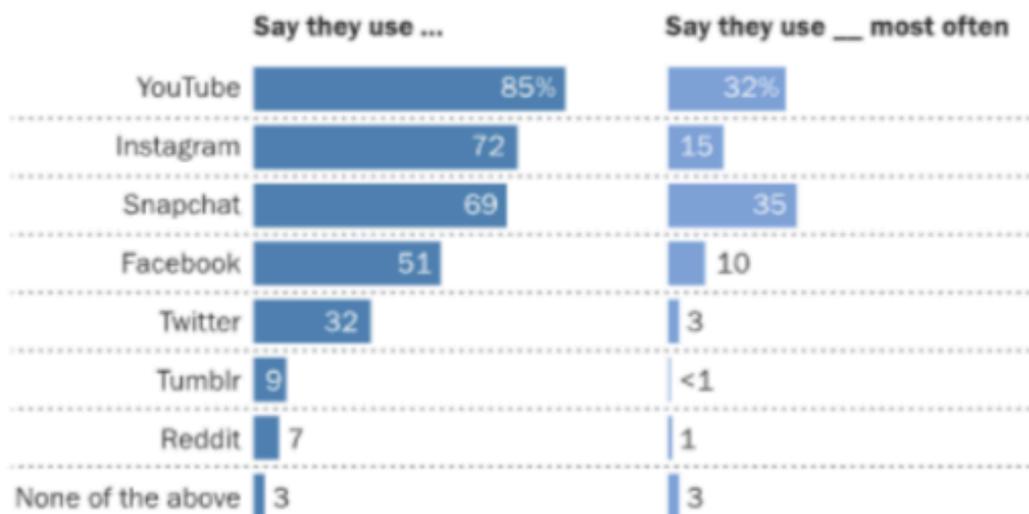
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- What is the difference b/w 2 charts?
- Why do the % add up to more than 100 in the first graph?
- Second graph gives us a *distribution* of teens over the different categories

Not equally likely outcomes

YouTube, Instagram and Snapchat are the most popular online platforms among teens

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"Teens, Social Media & Technology 2018"

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1. What is the chance that a randomly picked teen uses FB most often?
2. What is the chance that a randomly picked teen did *not* use FB most often?
3. What is the chance that FB or Twitter was their favorite?
4. What is the chance that the teen used FB, just not most often?
5. Given that the teen used FB, what is the chance that they used it most often?

Terminology

- **Experiment**: action that results in exactly one of several possible outcomes or results, and chance or randomness is involved - that is, each time we perform the action, the outcome will be different, and we don't know exactly which outcome will occur.
- An **event** is a collection of outcomes.
- A collection of all possible outcomes of an action is called a **sample space** or an **outcome space**. Usually denoted by Ω (sometimes also by S).
- An event is *always* a subset of Ω . Suppose we call the event A , then we write this as $A \subset \Omega$
- A **distribution** of the outcomes over some categories represents the proportion of outcomes in each category (each outcome appears in one and only one category)
- The **complement** of an event A is an event consisting of all the outcomes that are not in A . It is denoted by A^C and we have that $P(A^C) = 1 - P(A)$.

So far:

- If all the possible outcomes are *equally likely*, then each outcome has probability $1/n$, where $n = \#(\Omega)$
- Let $A \subseteq \Omega$, $P(A) = \frac{\#(A)}{\#(\Omega)}$
- Probabilities as proportions
- Sum of the probabilities of all the distinct outcomes should add to 1
- $0 \leq P(A) \leq 1, A \subseteq \Omega$
- A *distribution* of the outcomes over different categories is when each outcome appears in one and only one category.